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System Identification Using Transfer Matrix

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Abstract— **A new approach is proposed for multivariable system identification in the deterministic model framework. In the proposed approach, MIMO system is represented using transfer function (TF) matrix whose elements are the standard, fixed structure TFs like FOPDT, SOPDT etc. These model structures are capable of well approximating very large class of systems found in practice. The system identification problem is then considered as the problem of simultaneously estimating the parameters of all TFs in the TF matrix. This is posed mathematically as the constrained optimization problem, which minimizes the error between simulated and actual response. A genetic algorithm is used to solve the proposed optimization problem. The proposed approach is tested on several benchmark system identification test data sets. Results for two DaISy benchmark data sets, SISO example of flexible robotic arm and a MIMO example of an industrial dryer are discussed.**

Keywords*— *System Modeling; System Identification; Dynamics; Genetic Algorithm; Transfer Function Matrix; Flexible Robotic Arm; Industrial Dryer

I. INTRODUCTION

Mathematical models are quite useful in advanced engineering applications for design optimization, operational optimization, control, and fault diagnosis etc. System identification deals with the problem of obtaining models from the input output data. The most traditional system identification techniques are prediction error method (PEM) and the instrument variable method (IVM). These methods use the so-called black box model structures of the canonical forms e.g. ARX, ARMAX etc [1,2,7,8]. Though, these traditional identification techniques offer good solution to many real-life systems, they have certain issues like 1) difficulty in determining the model structure, and 2) numerical reliability due to need of solving the multidimensional nonlinear optimization problem in PEM case or system of linear equations in IVM case [1,2,7,8].

In last decade, these issues are addressed by the novel approach of subspace based MIMO linear state space (SS) model identification [5,6,7,8]. The subspace identification techniques have become popular because they are computationally fast and effective for the large dimensional MIMO systems. The main features of the subspace identification approaches are -- a) they require to determine only one structural parameter (state space model order) due to avoidance of the canonical forms, b) they are computationally fast, particularly in case of large MIMO systems, c) they are numerically robust because they are based on the numerically stable and reliable QR or LQ factorization and SVD algorithms, instead of numerically less reliable nonlinear optimization algorithms, and d) they have been found to yield good model accuracy for large and complex interactive MIMO systems [5,6,7,8].

However, subspace based state space identification approach also has certain limitations: 1) Loss of physical insight due to the state space model, 2) requirement of large amount of data to obtain acceptably accurate model, 3) though it is beneficial that it requires only one state space model structural parameter, there is a lack of any well established rule to choose this parameter, except not-so-matured methodology of rank testing, 4) state space parameters depend on data in a rather complicated way making it difficult to analyze and optimize the performance of the estimator, and 5) state space approach does not use quantitative optimization algorithms and the attempts to use optimization algorithms have had only limited success due to complicated mapping from data to estimated transfer function [9].

Several generic and user-friendly tools are available for model identification. For example, System Identification Toolbox of MATLAB [3,4] that contains various techniques for identifying time-series and state space, non-parametric and parametric models, is a popular identification tool among researchers and academicians. However, still such tools are difficult to use for process/control engineers in industry, who generally do not have academic training in system identification. Also, the practicing process/control engineers are more comfortable with transfer function models, in general. To fulfill the requirements of practicing engineers, recently there have been several attempts of proposing system identification based on the transfer function approach. For example, direct polynomial form of TF identification [11], using Laguerre series approximation of the transfer functions [12], or direct frequency domain identification [13].

In this paper, we propose a different approach to identify LTI multivariable systems in the form of MIMO transfer function matrix. The proposed approach not only addresses the aforesaid issues of the practicing engineers, but also provides an additional facility wherein any a priori knowledge about the system/process and model parameters additionally provides the practicing engineers a facility to utilize any a priori known system/process related information.

The paper is organized as follows. In section-2, some background of the genetic algorithms is given in brief. In section-3, the fixed structure transfer function model structures and the proposed identification approach are discussed. An algorithm is also given in brief. In section-4, simulation results are discussed. Section-5 contains the concluding remarks.

II. GENETIC ALGORITHMS

Genetic algorithms (GA) is one of the efficient, popular and proven evolutionary techniques for solving (global) optimization problems. The GA differs from other search techniques [14-17] by the use of concepts taken from natural genetics and evolution theory. The algorithm works with a population of strings, searching many peaks in parallel. By employing genetic operators it exchanges information between the peaks, hence reducing the possibility of terminating at a local optima thereby avoiding missing out of the global optimum. Genetic algorithms differ substantially from other optimization methods [14-17] in the following ways.

1) GAs search from a population of points, not a single point, so the search for the optimum is driven from many places in the search space simultaneously. This gives a better chance of finding the global optimum.
2) They work with a coding of the parameter set, not the parameter themselves.
3) They use the objective function directly, not the derivatives or other auxiliary knowledge. This objective function describes the goodness of the particular function. Being non-gradient-based technique, it is applicable even to the problems in which the objective functions are non-differentiable.
4) They use probabilistic transition rules to converge to the global optimum of a function. The rules are based on the natural idea of supporting good strings with higher fitness and removing poor strings with lower fitness. The best strings representing the best solutions are allowed to survive the evolution process with a higher probability.

The survival of the fittest and the death of the poor code strings are achieved by applying three basic operations: Reproduction, Crossover, and Mutation. The population of strings represents all the strings that are being processed in the current step of GA. The sequence of reproduction, crossover, and mutation generates a new population of strings from the previous population. The GA has been found very efficient and reliable in computing the global optimum, particularly in case when the objective function is observed to have multiple local optima. Hence, they have found a wide spread applications in solving the (global) optimization problems in many areas like pattern recognition applications, robotics and artificial life applications, expert system applications, system identification and so on [14-17].

III. PROPOSED APPROACH

A. *Proposed* Model *Structures*

Typically, the model structures used by the available identification techniques are classified as the non-parametric (e.g. ARX, ARMAX etc. time series models) or the parametric (e.g. State Space, Box Jenkins etc.) [1-7]. Some researchers have also tried to use transfer function approach to system identification [10-13]. However, one of the main difficulties of most system identification approaches is to determine the order of the model structures to use for a particular system.

In this work, we propose to use the popular and easy to use fixed structure transfer function models, such as FOPDT, SOPDT etc. The main reasons to choose these model forms and structures are –

1) These are the most commonly used transfer function model forms for the control applications.
2) Most (more than 90%) of the physical systems can be approximated (about 60% by FOPDT & 30% by SOPDT models) using these forms with acceptable accuracy, in the region of interest.
3) The models being of lower order are computationally cheap, and also suitable for online control applications like tuning of PID controller, for use as the model in model based control techniques.
4) The transfer function models are more intuitive, known to practicing engineers. The practicing engineers based on the process knowledge and experience can easily determine these structures. Hence the identification approach based on these structures can become more popular and simple to use in industry.

The mathematical representations of the fixed structure transfer function models considered are well known as given below:

The standard *First-Order-Plus-Dead-Time or FOPDT* transfer function model is mathematically represented as –

$$G(s)=\frac{Ke^{-Ls}}{Ts+1} \qquad (1)$$

The standard *Two-Time-Constant-Plus-Dead-Time* or *2TCPDT* transfer function structure covers two time constant in series configuration (i.e. it is over damped 2nd order transfer function). It is mathematically represented as –

$$G(s)=\frac{Ke^{-Ls}}{(T_1s+1)(T_2s+1)} \qquad (2)$$

The standard *Second-Order-Plus-Dead-Time or SOPDT* transfer function structure covers the under damped system response and is mathematically represented as --

$$G(s)=\frac{Ke^{-Ls}}{(T_n^2s^2+2zT_ns+1)} \qquad (3)$$

The standard 2-*Time-Constant-1-Zero-Plus-Dead-Time* or *2TC1ZPDT* transfer function model structure covers the inverse response observed in many industrial systems and is mathematically represented as --

$$G(s) = \frac{K\,(T_z s + 1)e^{-Ls}}{(T_1 s + 1)(T_2 s + 1)} \qquad (4)$$

Where, K – Process Gain

T, T_1, T_2, T_z = Process Time Constants(zero/pole)

T_n – Natural Period of Oscillation

z – Damping constant, and

L – Dead time (Time delay)

B. Proposed Algorithm

In the proposed approach, the element transfer functions of the transfer matrix can be any of the various standard and known transfer function structures like *FOPDT, 2TCPDT, SOPDT, 2TC1ZPDT* as described above. The fixed structure transfer function model identification problem can be posed as the problem of estimating the model parameters (like K, T_i, T_n, z and L). The transfer function parameter estimation problem is posed mathematically as the *constrained optimization (minimization)* problem [14, 15]. The objective function to be minimized is a function of the simulation error i.e. some function of error between model simulated output $\hat{y}(t)$ and the measured output y(t). This constrained optimization problem can be represented as given in (5).

$$\min_{\theta}\{f(y-\hat{y})\} \quad s.t.\ \theta_{\min} \le \theta \le \theta_{\max} \qquad (5)$$

Where θ= {K, Ti, T_n, z, L}

In (5), a set of the decision variables is actually a set of parameters to be estimated i.e. θ = {K, Ti, T_n, z, L}, whereas θ_{min} and θ_{max} are lower and upper constraints (bounds) on the parameters to be estimated. The constrained optimization formulation, in fact, allows the user to exploit any a priori knowledge about the possible range of the model parameters, or to enable/disable some input-output channels in MIMO identification.

Now we describe an algorithm for the proposed approach of system identification.

Algorithm: Fixed Structure Transfer Matrix Identification Algorithm (FSTMIA)

Inputs:

1. Set of measured input-output data.
2. Constraints on the parameters θ_{min}, θ_{max}

Outputs:

A set of estimated parameters θ.

Algorithm:

Begin

1. Choose suitable values for the parameters θ within the prescribed constraints.
2. Simulate the output response using the input data set and the trial parameter set.
3. Compute the objective function using the error between the measured and the simulated model output.
4. If the minimum of the error function is *not* reached, go back to step-1. Otherwise go to next i.e. step-5.
5. Terminate the algorithm with the resulted parameter set where the minimum of the error function is obtained, along with the minimum value of the error function achieved.

End.

In the proposed approach this constrained optimization problem is solved using *genetic algorithm (GA)* [14-17], which is a popular and proven optimization technique that is observed to reliably find the global optimum solution. The choice of the decision variable values (i.e. the parameter set θ) in step-1 are iteratively decided by the basic operations of the GA technique e.g. reproduction, mutation and crossover. The optimum solution of the decision variables (i.e. the parameter set θ_{opt}) is the best candidate obtained as per 'the survival of the fittest' evolutionary concept of GA. In general, the (global) optimum is obtained when GA is terminated according to some appropriate termination criterion [15]. Thus, the parameters of the identified model are actually the optimum solution obtained by GA during the parameter estimation step.

IV. EXAMPLES

The proposed system identification approach (algorithm) is then programmed in MATLAB [4] and then is tested using several benchmark test data sets for the system identification. Results for two benchmark examples from DaISy [18], a SISO example of the robot arm, and a MIMO example of an industrial dryer are discussed here. The performance measure used to judge the fitment of the model is the % error between the original output and the output simulated by the deterministic part of the model [19], which is defined as --

$$\%\,Error = \frac{100}{p}\sum_{i=1}^{p}\sqrt{\frac{\sum_{j=0}^{N-1}(y_i(j)-\hat{y}_i(j))^2}{\sum_{j=0}^{N-1}(y_i(j))^2}} \qquad (6)$$

Where,

p = number of outputs, N = total number of data points,

$y_i(j)$ = measured value of output i at j^{th} instant t,

$\hat{y}_i(j)$ = model simulated value of output i at j^{th} instant t,

Example-1: Consider a SISO system of a flexible robotic arm installed on an electric motor. The input variable is the measured reaction torque of the structure on the ground and the output variable is the acceleration of the robot arm. This benchmark data set is taken from DaISy [18], which consists of 1024 input-output data points obtained by applying a periodic sine sweep on the experimental setup established in the laboratory of production manufacturing and automation at Katholieke Universiteit Leuven, Belgium. We applied the proposed approach on this data set taking first 500 data points for identification, and the remaining 524 data points for the validation. The performance measures obtained by various model structures are given in Table-1. Among them, the SOPDT structure is found to provide the best-fit model. The simulated and actual response of this model for identification and validation data sets shows (refer Fig. 1) that both the responses match very well. Fig. 2 shows the enlarged plot for the data points after first 250 data points in each case (i.e. identification and validation) for clarity. It is found that the identified model matches well.

TABLE 1: SIMULATION ERROR BY THE PROPOSED APPROACH FOR DIFFERENT MODEL STRUCTURES FOR EXAMPLE-1

Approach	Model Order	Identification	Validation
Proposed FSTMIA Approach	FOPDT	98.46	98.24
	2TCPDT	98.45	98.23
	2TC1ZPDT	98.45	98.24
	SOPDT	***35.90***	***34.75***

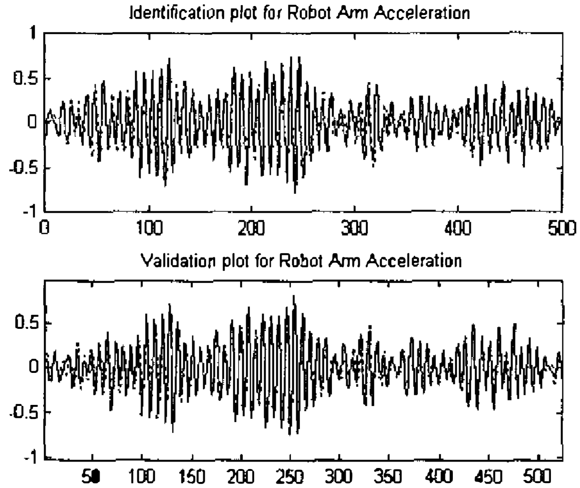

Figure 1: Identification and Validation response plots of the best model in Example-1: measured (solid), proposed (dotted)

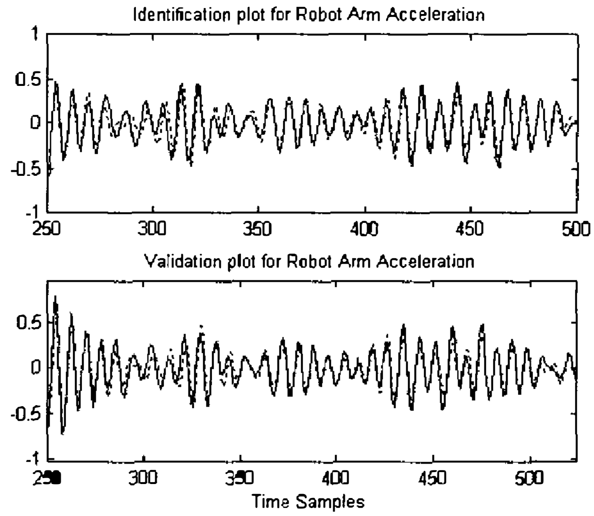

Figure 2: Enlarged Identification and Validation response plots of the best model in Example-1: measured (solid), proposed (dot) (For clarity, this enlarged plot shows the data points after first 250 data points in each case.)

Example-2: Consider a MIMO system of an industrial dryer. The system has three inputs: fuel flow rate, hot gas exhaust fan speed, and rate of flow of raw material, and three outputs: dry bulb temperature, wet bulb temperature, and moisture content of the raw material leaving the dryer. The first two inputs are manipulated variables, and are changed by applying PRBS signal, whereas the third input is disturbance, but can be measured. The pre-treated data for this system is available as a benchmark data at DaISy [18], which consists of 867 data points with a sampling period of 10 seconds. We used the first 600 data points for identification and remaining 267 data points for the validation.

The selected identification data set is used to identify state space model using recently popular approach of sub-space identification algorithm (N4SID) available in the identification toolbox of MATLAB [3,4], and also to identify transfer (function) matrix model using the proposed *fixed structure transfer matrix identification algorithm* (FSTMIA). The % simulation error obtained by various model structures, and by these approaches is compared in Table-1. It is observed that the best-obtained model with N4SID algorithm is a 6-state state space model.

TABLE 2: SIMULATION ERROR BY DIFFERENT APPROACHES AND MODEL STRUCTURES

Algorithm	Order	Identification	Validation
N4SID algorithm MATLAB [3,4]	2	87.82	87.82
	4	39.30	69.06
	6	***37.02***	***58.78***
proposed FSTMIA Approach	FOPDT	37.02	58.78
	2TCPDT	39.99	56.44
	2TC1ZPDT	33.03	53.99
	SOPDT	75.94	86.00
Algorithm 10.5 in [19]	2	31.90	59.72
	4	30.66	60.00
	6	30.72	60.20
	8	***29.72***	***58.22***

Whereas the best model obtained by the proposed FSTMIA algorithm is *2TC1ZPDT model i.e. 2-time-constant-1-zero-plus-dead-time* as given by equation (3). In Table-1, the results obtained by the algorithm 10.5 in [19] are also given, for which 8^{th} order model is the best-fit model. It is observed that % error for identification data set is minimum in case of algorithm 10.5 of [19]. However, proposed approach shows the minimum % simulation error for the validation data. Hence, the model identified by the proposed approach can be considered better, because it generalizes better, in comparison to the other approaches, on the validation data set.

The actual and simulated responses for identification and validation data sets (see Figs-3 to 5) also confirms that the model identified by the proposed approach are more accurate.

The proposed approach is tested on several other benchmark datasets, and on actual industrial data and is found accurate. These results are not given here due to the space constraint.

V. CONCLUDING REMARKS

The proposed approach is novel in a way that the system identification problem is posed as a fixed structure transfer (function) matrix parameter estimation problem that is solved as a constraint optimization problem using genetic algorithm. The proposed approach is simple to use and can be successfully applied for system identification, obviously to the limitation of the chosen model structure. The model identified by the proposed approach is found to be user-friendlier, particularly for the practicing process engineers in industry, and also more accurate compared to those obtained by the popular sub-space algorithm available in the identification toolbox of MATLAB [3,4] and also compared to the results published earlier in the literature [19].

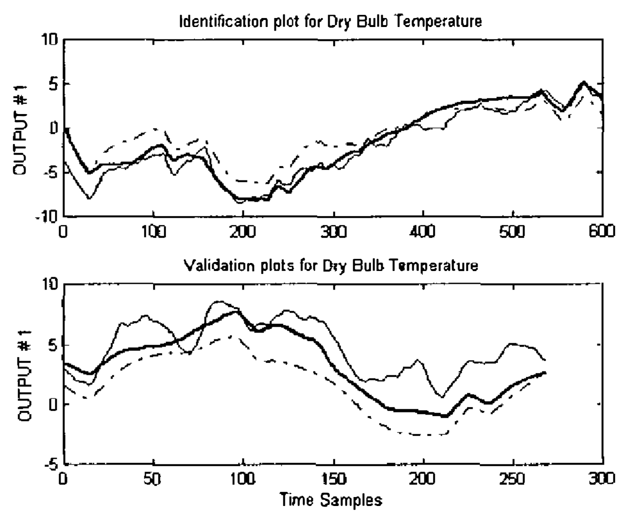

Figure 3: Identification and Validation response plots of the best models in Example-2 for output-1: measured (thin solid), proposed (thick solid), subspace N4S6 (dash-dot).

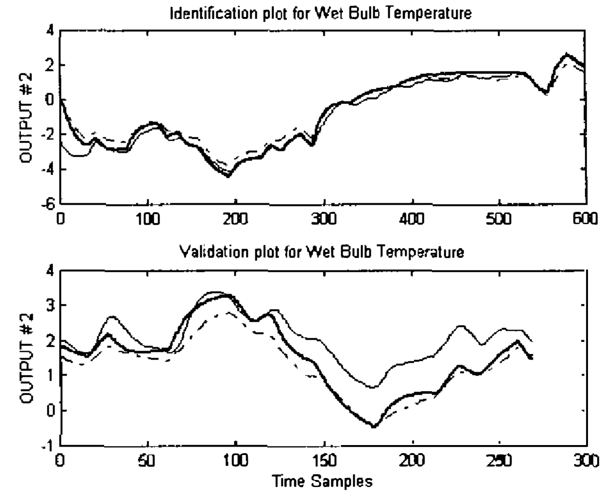

Figure 4: Identification and Validation plots of the best models in Example-2 for output-2: measured (thin solid), proposed (thick solid), subspace N4S6 (dash-dot).

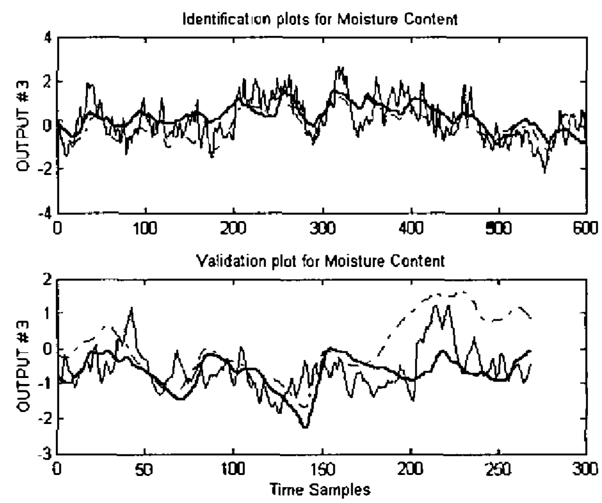

Figure 5: Identification and Validation plots of the best models in Example-2 for output-3: measured (thin solid), proposed (thick solid), subspace N4S6 (dash-dot).

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