Chapter 3
Reliability

3.1 Introduction

Offering warranty results in additional costs to the manufacturer due to the servicing of claims resulting from product failures under warranty. Product failures are depend on product reliability and this, in turn, is influenced by several factors, some under the control of the manufacturer (decisions made during the design and production phases) and others under the control of the customer (operating environment, usage mode and intensity, and so forth).

During the design phase, an assessment of product reliability is made based on product design and available knowledge of component reliability (often supplied by vendors). This, in combination with limited test data collected during the development phase, forms the basis for deciding whether or not to launch the product. This decision must be made at an early stage because building in reliability is costly but the consequence of not having adequate reliability can be costlier (due to higher warranty costs, product recall, etc.) Warranty data provide a valuable source of information for assessing the reliability of an item in operation (called the "field reliability") and to make decisions regarding the reliability improvements needed to control the consequences of unreliability.

A good understanding of reliability theory is essential for designing proper systems for the collection and analysis of warranty data. These provide essential information for making effective management decisions. In this chapter, we briefly discuss some topics from reliability theory that will be used in later chapters.

The outline of the chapter is as follows. We begin with a brief discussion of some basic concepts in Sect. 3.2. It is important that product reliability be viewed from a product life perspective. This is discussed in Sect. 3.3, where we consider the life cycle of both standard and custom-built products. This provides a framework for characterization of the different notions of product reliability that are discussed in Sect. 3.4. Reliability modeling is important for a variety of reasons, including estimation of reliability based on parametric models, and prediction of warranty

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costs. These issues are discussed further in later chapters of the book. Section 3.5 looks at the modeling process in general. The modeling of first failure needs to done differently from that of subsequent failures, since the latter depend on actions taken to rectify failures. Sections 3.6 and 3.7 deal with the modeling of first and subsequent product failures, respectively. Even a simple product is comprised of several components. In Sect. 3.8, we discuss the linking of product reliability to component reliabilities. Finally, in Sect. 3.9, we look at the relationship between warranty and reliability. The use of reliability models to predict warranty costs is discussed in Chaps. 6 and 7.

3.2 Basic Concepts

3.2.1 Product Deterioration

All products degrade with age and/or usage. When product performance falls below a desired level, the product is deemed to have *failed*. Failures occur in an uncertain manner and are influenced by factors such as design, manufacture or construction, maintenance, and operation. In all of these, the human factor is an important element.

Failure is often a result of the effect of deterioration. The deterioration process leading to a failure is a complicated process that varies with the type of product and the material used. The rate at which deterioration occurs is a function of time and/or usage intensity.

3.2.2 Fault

A *fault* is the state of the system characterized by its inability to perform its required function. (Note: This excludes situations arising from preventive maintenance or any other intentional shutdown period during which the system is unable to perform its required function.) A fault is therefore a state resulting from a failure.

It is important to differentiate between failure or fault and error. The International Electrotechnical Commission defines an error to be a "discrepancy between a computed, observed or measured value or condition and the true, specified or theoretically correct value or condition." [7] As a result, an error is not a failure, because it is within the acceptable limits of deviation from the desired performance (target value). An error is sometimes referred to as an incipient failure [19].

3.2.3 Failure Modes

A *failure mode* is a description of a fault. It is sometimes referred to as fault mode (for example, in [7]). Failure modes are identified by studying the performance of the item. A classification scheme for failure modes is shown in Fig. 1.7 of [2] and a brief description of the different failure modes is as follows:

1. *Intermittent failures*: Failures that last for only a short time. A good example of this is a switch that sometimes does not make proper contact.
2. *Extended failures*: Failures that continue until some corrective action rectifies the failure. These can be divided into the following two categories:

   – *Complete Failures* which result in total loss of function.
   – *Partial Failures* which result in partial loss of function.

   Each of these can be further subdivided into the following:

1. *Sudden failures*: Failures that occur without any warning.
2. *Gradual failures*: Failures that occur with signals to warn of the occurrence of a failure.

A complete and sudden failure is called a *catastrophic failure* and a gradual and partial failure is designated a *degraded failure*.

3.2.4 Failure Causes and Classification

According to IEC 50 (191), *failure cause* is "the circumstances during design, manufacture or use which have led to a failure". Failure cause is useful information in the prevention of failures or their reoccurrence. Failure causes may be classified based on the causes of failure as follows:

1. Design Failure: Due to inadequate design.
2. Weakness failure: Due to weakness (inherent or induced) in the system so that the system cannot stand the stress it encounters in its normal environment.
3. Manufacturing failure: Due to non-conformity during manufacturing.
4. Aging failure: Due to the effects of age and/or usage.
5. Misuse failure: Due to misuse of the system (operating in environments for which it was not designed).
6. Mishandling failures: Due to incorrect handling and/or lack of care and maintenance.

3.2.5 Failure Mechanism

According to IEC 50 (191), a *failure mechanism* is "the physical, chemical or other processes that may lead to a failure". There are other causes as well, such as human errors.

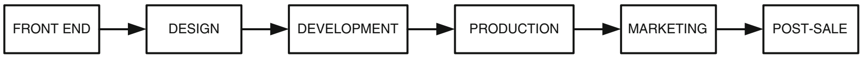

Fig. 3.1 Product life cycle (standard product)

Mechanisms of failure can be divided into two broad categories, (1) overstress mechanisms, and (2) wear-out mechanisms [6]. In the former case, an item fails only if the stress to which the item is subjected exceeds the strength of the item. If the stress is below the strength, the stress has no permanent effect on the item. In the latter case, however, the stress causes damage that usually accumulates irreversibly. The accumulated damage does not disappear when the stress is removed, although sometimes annealing is possible. The cumulative damage does not cause any performance degradation as long as is it below the endurance limit. Once this limit is reached, the item fails. The effects of stresses are influenced by several factors—geometry of the part, constitutive and damage properties of the materials, manufacturing, and operational environment.

3.3 Product Life Cycle

The life cycle of a product is basically the period of time during which it is in existence, either conceptually or physically, and may be defined in various ways. Below we look at the product life cycles for standard and custom-built products. These differ somewhat, and both depend on the point of view taken—buyer, manufacturer, seller, and so forth.

3.3.1 Standard Products

A product life cycle for a standard consumer durable or an industrial product, from the point of view of the manufacturer, is the time from initial concept of the product to withdrawal of the product from the marketplace. The life cycle involves several stages, as indicated in Fig. 3.1.

The process begins with the idea of building a product to meet some customer requirements, such as performance targets, including reliability. This is usually based on a study of the market and the potential demand for the product being planned. The next step is to carry out a feasibility study. This involves determining if it is possible to achieve the targets within specified cost limits. If this analysis indicates that the project is feasible, an initial product design is undertaken. A prototype is then developed and tested.

It is not unusual at this stage to find that achieved performance levels of the prototype product are below the target values. In this case, further product

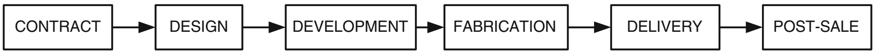

Fig. 3.2 Product life cycle (custom built product)

development is undertaken to overcome the problem. Once this is achieved, the next step is to carry out trials to determine performance of the product in the field and to start a pre-production run. This is required because the manufacturing process must be fine-tuned and quality control procedures established to ensure that the items produced have the same performance characteristics as those of the final prototype.

After this, the production and marketing efforts begin. The items are produced and sold. Production continues until the product is removed from the market because of obsolescence and/or the launch of a new product. Post-sale support of the product continues at least until expiration of the warranty on the last item sold, but can continue beyond this point in terms of spare parts, service contracts, etc.

3.3.2 Custom Built Products

The life cycle for a custom built product is slightly different and is as shown in Fig. 3.2. Here the product requirement is supplied by the customer and then jointly agreed upon by the customer and manufacturer. The manufacturer builds the product to these specifications under a negotiated contract. The process then follows basically the same steps as those for standard products.

3.4 Product Reliability

3.4.1 Concept and Definition

Reliability of a product conveys the concept of dependability, successful operation or performance, and the absence of failures. It is an external property of great interest to both manufacturer and consumer. Unreliability (or lack of reliability) conveys the opposite. More technical definitions of reliability are the following:

1. The ability of an item to perform a required function, under given environmental and operational conditions and for a stated period of time. [8]
2. The reliability of a product (system) is the probability that the product (system) will perform its intended function for a specified time period when operating under normal (or stated) environmental conditions. [4]

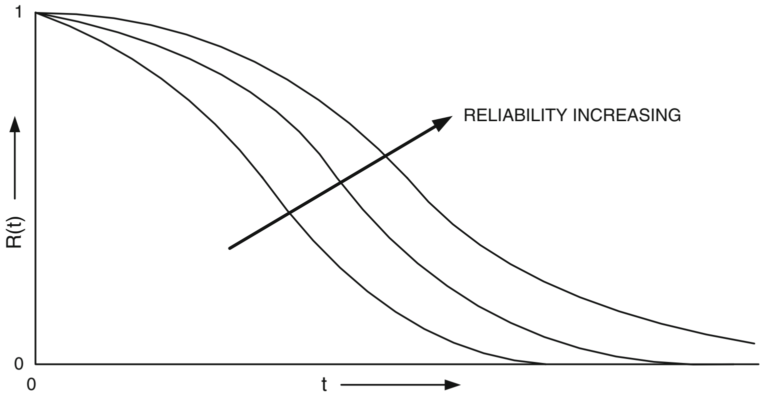

Fig. 3.3 Plots of reliability functions

The reliability of product is given by a function $R(t)$ with the following properties:

1. $R(t)$ is a non-increasing function of t, $0 \le t < \infty$
2. $R(0) = 1$ and $R(\infty) = 0$

Typical plots of $R(t)$ are shown in Fig. 3.3.

3.4.2 Product Life Cycle Perspective

From a product life cycle perspective, there are several different notions of reliability. Figure 3.4 [13] shows how these are sequentially linked and the factors that affect them. We briefly discuss four reliability concepts.

3.4.2.1 Design Reliability

At the design stage, the desired product reliability is determined through a tradeoff between the cost of building in reliability and the consequences of failures. This trade-off is discussed in detail in [13]. From this, one derives the reliability specification at the component level. One then evaluates the design reliability.[1]

3.4.2.2 Inherent Reliability

For standard products produced in volume, the reliability of the produced item can differ from the design reliability because of assembly errors and component

[1] The linking of component reliabilities to product reliability is discussed in Sect. 3.8.

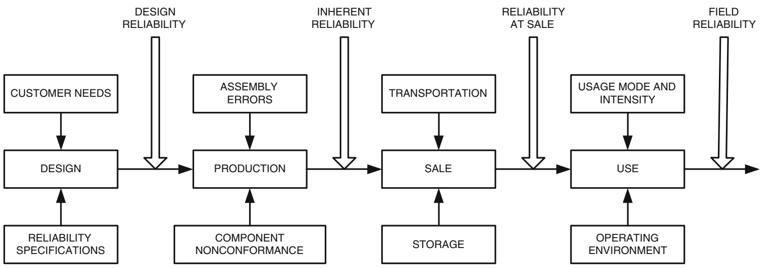

Fig. 3.4 Different notions of reliability (standard product)

non-conformance. The reliability of produced items is the "inherent reliability" of the product.

3.4.2.3 Reliability at Sale

After production, the product must be transported to the market and is often stored for some time before it is sold. The reliability of a unit at sale depends on the mechanical load (resulting from vibrations during transport) and impact load (resulting from mishandling) to which it has been subjected, the duration of storage, and the storage environment (temperature, humidity, etc.). As a result, the reliability at sale can differ from the inherent reliability. Once an item is sold, it may either be stored for an additional time (if the unit has been purchased for later use or is used as a spare), or it may be put into operation immediately. The additional storage time may again affect the reliability of the unit.

3.4.2.4 Field Reliability

The reliability performance of a unit in operation depends on the length and environment of prior storage and on operational factors such as the usage intensity (which determines the load—electrical, mechanical, thermal, chemical—on the unit), usage mode (whether used continuously or intermittently), and operating environment (temperature, humidity, vibration, pollution, etc.) and, in some instances, on the human operator. The reliability performance of an item in operation is often referred to as "field reliability."

Example 3.1 Washing machines are designed to some nominal functional and reliability requirements. The functional requirements might be, for example, a nominal load of 12 pounds per wash and a usage intensity of 6 washes per week. The reliability requirement might be, for example, that not more than one washer per thousand fails in the first year when the machine is operated under normal load and usage intensities. This defines the design reliability.

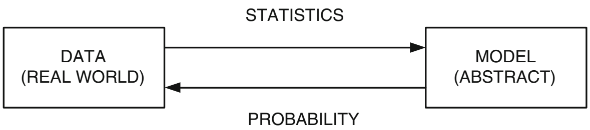

Fig. 3.5 Link between real world and model

Due to variations in manufacturing, the inherent reliability can differ from the design reliability. If some of the bearings are defective, for example, they can wear faster, causing washing machines with these defective bearings to fail earlier.

In practice, the load will vary from wash to wash. If the load per wash is significantly greater than 12 pounds, then it can affect the performance and the reliability of components such as bearings, motor, etc. This may also occur if the usage intensity is significantly higher than the nominal value.[2]

3.5 Models and Modeling Process

3.5.1 The Role of Models

Models play an important role in solving a variety of problems. A model is a representation of the real world that is relevant to the problem. There are many different types of models. Some of these are physical models and others abstract. We confine our attention to mathematical models.[3]

A mathematical model is an abstract representation involving a mathematical formulation. When uncertainty is a significant feature of the real world (as is the case, for example, in the time to failure of an item), then concepts from probability theory and statistics, as well as data from the real world, play an important role in linking the model to reality, as indicated in Fig. 3.5.[4]

3.5.2 Modeling Process

Building a model is an iterative process involving several steps, as indicated in Fig. 3.6.[5]

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[2] Note that for some products of this type designed for domestic use, the warranty becomes null and void if used in a commercial context (e.g., in a laundromat).

[3] There are many books that discuss models. See for example, [11] and the references cited therein.

[4] In this chapter, we confine our attention to models for product failures. In later chapters we deal with models for other purposes, such as estimating warranty costs, etc.

[5] There are many books that discuss the modeling process in detail; see for example [11] and the references cited therein.

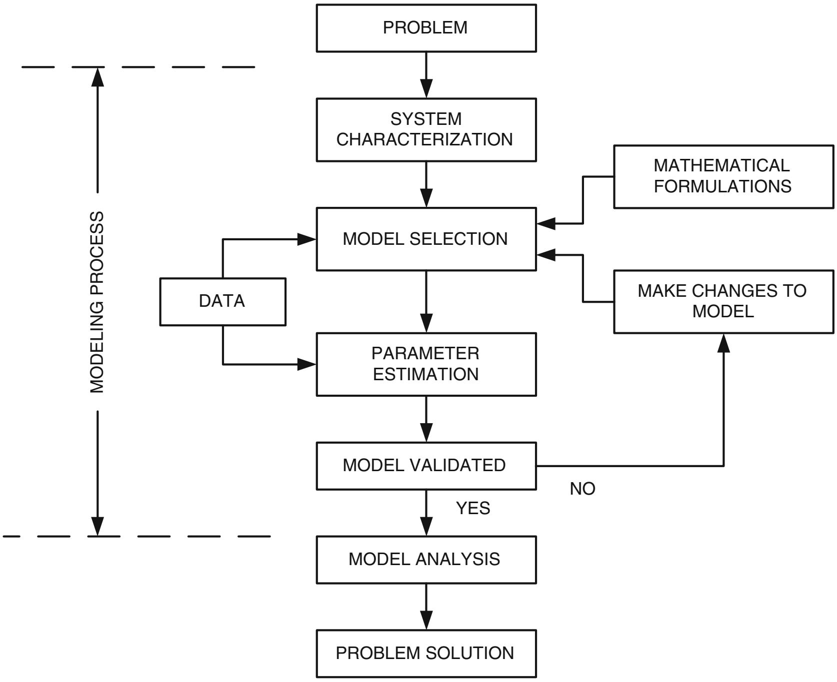

Fig. 3.6 Modeling process

In the following, we discuss the key steps in the modeling process. These principles will be applied to reliability modeling in the following section.

Step 1: Defining the Problem

Problem definition depends on the context. In this chapter, the problem is to predict product failures over time.

Step 2: System Characterization

Characterization of a system details the salient features of the system that are relevant to the problem under consideration. This generally involves a process of simplification. The variables used in the system characterization and the relationships between them are problem dependent. If the problem were to understand product failures, then the system characterization would involve reliability theory; if the problem were to study the impact of warranty on sales, then one would use theories from marketing; and so forth. The characterization of the cause–effect relationship between the variables can be done in several ways. A common approach is to use diagrams with nodes representing variables and directed arcs indicating the cause–effect relationships.

Step 3: Model Selection

There are two approaches to model selection. These are:

Empirical (black-box) Approach: Model selection is based solely on the data available.

Physics-based (white-box) Approach: Model selection is based on relevant theories (for example, the different theories for component failures).

The kind of mathematical formulation to be used depends on the system characterization and the approach used. For modeling product failures based on the black-box approach, distribution functions are used to model the time to first failure and counting processes are used to model subsequent failures.

Step 4: Parameter Estimation

The model will involve one or more unknown parameters, and numerical values for these are needed. These are obtained by means of a statistical methodology called parameter estimation. The approach used depends on the type and amount of data available. This is discussed in Chap. 9.

Step 5: Model Validation

Validation involves testing whether or not the model selected (along with the assigned parameter values) models the real world sufficiently adequately to yield a meaningful solution to the problem of interest. The approach used can vary from a visual comparison between model predictions and observed data to statistical methods such as hypothesis testing and goodness-of-fit. These procedures are discussed in Chap. 10.

Step 6: Model Analysis

One can use several different approaches to analysis of the model. These include analytical methods (which yield closed form results as functions of the model parameters), computational methods, and simulation.

3.6 Modeling First Failure and Reliability

3.6.1 Basic Results

Let T be a continuous random variable denoting the time to failure of an item. This is modeled by a *distribution function* $F(t;\theta)$ (also called a *cumulative distribution function* or *CDF*), which characterizes the probability that the item fails before t. The CDF is given by

$$F(t;\theta) = P\{T \le t\}. \tag{3.1}$$

Comment: For notational ease, the dependence on the parameter θ is often suppressed and $F(t)$ is used instead of $F(t;\theta)$. We follow this convention in the remainder of the chapter.

$F(t)$ is called the *failure distribution* function. When $F(t)$ is differentiable, the result is called the *failure density* function, and denoted $f(t)$. This is given by

$$f(t) = \frac{dF(t)}{dt}. \tag{3.2}$$

The *reliability* function $R(t)$ (sometimes denoted $\bar{F}(t)$),[6] is defined to be the probability that the item survives for at least a period t, so that

$$R(t) = P\{T > t\} = 1 - F(t). \tag{3.3}$$

The conditional probability that the item will fail in the interval $[t, t + \delta t)$, given that it has not failed prior to t, is given by

$$F(\delta t | T > t) = \frac{F(t + \delta t) - F(t)}{R(t)} \tag{3.4}$$

The *hazard function* (or *failure rate function*) $h(t)$ associated with $F(t)$ is defined as

$$h(t) = \lim_{\delta t \to 0} \frac{F(\delta t | T > t)}{\delta t} = \frac{f(t)}{R(t)} \tag{3.5}$$

The hazard function $h(t)$ can be interpreted as the probability that the item will fail in $[t, t + \delta t)$, given that it has not failed prior to t. In other words, it characterizes the effect of age on item failure more explicitly than $F(t)$ or $f(t)$.

The *cumulative hazard function*, $H(t)$, is defined as

$$H(t) = \int_0^t h(t')dt' \tag{3.6}$$

$H(t)$ is also called the *cumulative failure rate function.*

Appendix A provides a list of distributions that have been used extensively in reliability modeling.

Example 3.2 [Two-parameter Weibull Distribution] The two-parameter Weibull distribution is used extensively in reliability modeling. The CDF for this distribution is

$$F(t;\theta) = 1 - e^{-(t/\alpha)^\beta} \tag{3.7}$$

[6] We will use both notations throughout the book.

for $t \geq 0$. The parameter set is $\theta = \{\alpha, \beta\}$, with $\alpha > 0$ and $\beta > 0$. α is a scale parameter and β is a shape parameter. The failure density and hazard functions are given by

$$f(t;\theta) = \frac{\beta t^{(\beta-1)} e^{-(t/\alpha)^\beta}}{\alpha^\beta} \tag{3.8}$$

and

$$h(t;\theta) = \frac{\beta t^{(\beta-1)}}{\alpha^\beta} \tag{3.9}$$

The shape of the hazard functions depend on the shape parameter and can have one of the following three shapes:

1. Increasing failure rate (IFR) when $\beta > 1$
2. Decreasing failure rate (DFR) when $\beta < 1$
3. Constant failure rate when (CFR) $\beta = 1$.

Figure 3.7 shows plots of the density and hazard functions for $\beta = 0.5$, 1, and 2. These values of the shape parameter illustrate the three regions indicated above.

3.6.2 Design Reliability

Let $F_0(t)$ denote the design failure distribution. Let $R_0(t)$, $f_0(t)$ and $h_0(t)$, denote, respectively, the reliability function, the density function and the hazard function associated with $F_0(t)$. The hazard function $h_0(t)$ is IFR (curve A in Fig. 3.8),[7] which reflects the effect of ageing. Good design requires that the hazard function be below some specified value over the useful life of the product.

3.6.3 Effect of Quality Variations in Manufacturing

Two causes of variations are (1) assembly error and (2) component non-conformance.

3.6.3.1 Assembly Errors

Even a simple product consists of several components that are assembled in production. The type of assembly operation depends on the product. For an

[7] Figure 3.8 shows four plots (A–D). Plot A is the designed hazard function. Plots B–D indicate how this is affected as a result of quality variations, as discussed in the next subsection.

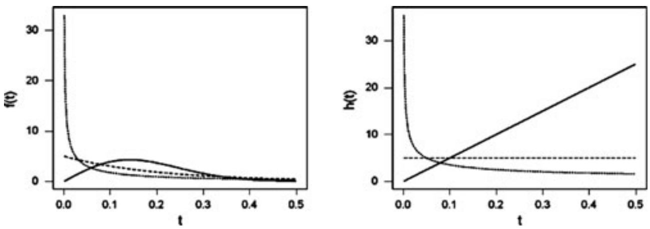

Fig. 3.7 Plots of Weibull density and hazard functions for $\beta = 0.5$, 1, and 2 (*top*, *middle*, and *bottom curves*, respectively, along left axis in both plots)

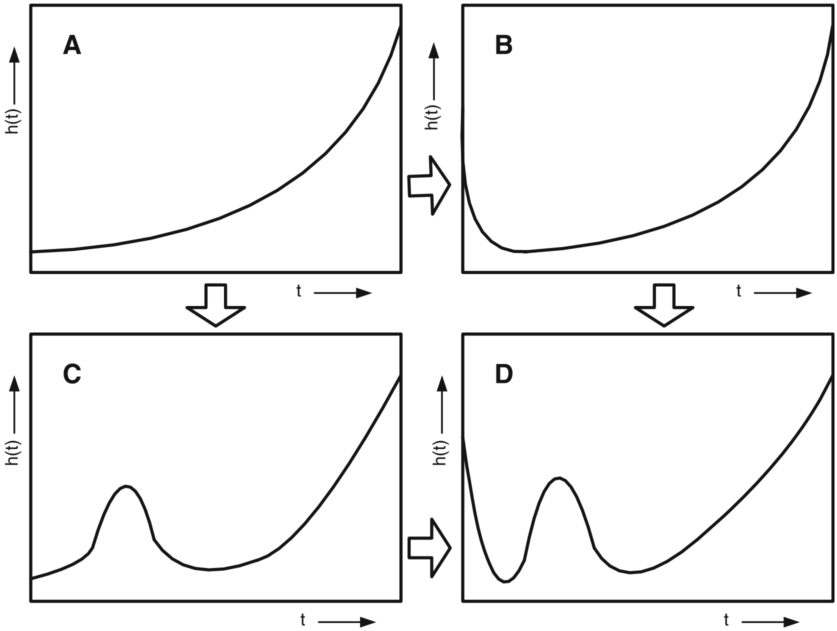

Fig. 3.8 Shapes of hazard function with quality variations

electronic product, one of the assembly operations is soldering. If the soldering is not done properly (called dry solder), then the connection between the components can break within a short period, leading to a premature failure. For a mechanical component, a premature failure can occur if the alignment is not correct or the tolerances are violated.

Failures resulting from assembly errors can be viewed as a new mode of failure that is different from other failure modes that one examines during the design process. Let $F_1(t)$ denote the distribution function associated with this new failure mode, and $R_1(t)$, $f_1(t)$ and $h_1(t)$ the survivor function, density function and failure rate function associated with $F_1(t)$. The failure rate $h_1(t)$ is a decreasing function of t, implying that failure will occur sooner rather than later, and that the mean time to failure (*MTTF*) under this new failure mode is much smaller than the design *MTTF*.

Not all items are affected by assembly errors. Let q, $0 \leq q \leq 1$, denote the probability that an item has an assembly error. The reliability of produced items can be modeled by a modified competing risk model [12] given by[8]

$$R_a(t) = R_0(t)[1 - qF_1(t)] \tag{3.10}$$

Comments: (1) If $q = 0$, then $R_a(t) = R_0(t)$. If $q = 1$, then $R_a(t) = R_0(t)R_1(t)$, which is the standard competing risk model (see Appendix A). (2) The hazard function $h_a(t)$ associated with $F_a(t)$ is the sum of the design hazard function (which is increasing) and the hazard function for the new failure mode (which is decreasing). As a result, $h_a(t)$ has a bathtub shape (curve B in Fig. 3.8).

3.6.3.2 Component Non-Conformance

Because of variations in quality, some components do not meet design specifications. Suppose, in particular, that their *MTTF* is much smaller than intended. Items that are produced with such nonconforming components will also tend to have an *MTTF* that is much smaller than the intended design value. To model this situation, we proceed as follows. Let $F_2(t)$ denote the failure distribution of items that have nonconforming components, and $R_2(t)$, $f_2(t)$ and $h_2(t)$denote, respectively, the survivor, density, and failure rate functions associated with $F_2(t)$. $h_2(t)$ is an increasing function of t, with $h_2(t) > h_0(t)$ for all t.

Let p, $0 \leq p \leq 1$, denote the probability that an item produced has nonconforming components, so that its failure distribution is given by $F_2(t)$. Then $(1 - p)$ is the probability that the item is conforming and has failure distribution $F_0(t)$. As a result, the reliability of the items produced is given by

$$R_n(t) = (1 - p)R_0(t) + pR_2(t) \tag{3.11}$$

Comments: (1) This is a standard mixture model involving two distributions (see Appendix A). If $p = 0$, then $R_n(t) = R_0(t)$, as to be expected, and if $p = 1$, then $R_n(t) = R_2(t)$, as all items have nonconforming components. (2) The hazard function $h_n(t)$ associated with $F_n(t)$ has an N-shape [increasing followed by decreasing and ultimately increasing (curve C in Fig. 3.8)].

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[8] [5] deals with this model, which they call the "general limited failure population model." They give an interpretation of the model in the context of reliability theory where an item failure is due to one of two competing causes—common cause and another, called special cause. The time to failure due to common cause failure (for example, wear-out) has a distribution function $F_0(t)$ and a proportion q can fail due to the other cause (for example, infant mortality) with a distribution function $F_1(t)$.

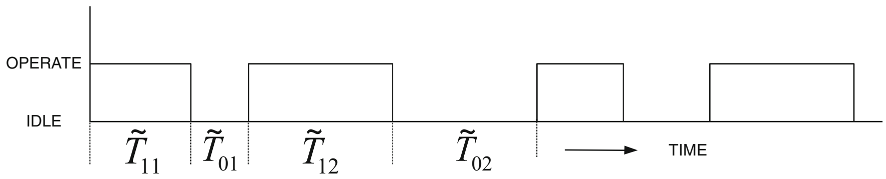


Fig. 3.9 Intermittent usage time history

3.6.3.3 Modeling the Combined Effect

With both assembly errors and component nonconformance may occur, the reliability of the items produced is given by

$$R_q(t) = [(1-p)R_0(t) + pR_2(t)](1 - qF_1(t)) \tag{3.12}$$

In this case, the hazard function $h_q(t)$ associated with $F_q(t)$ $[= 1 - R_q(t)]$ has a W-shape (curve D in Fig. 3.8).

Comment: The plots shown in Fig. 3.8 provide a basis for identifying quality variation problems from empirical plots of the hazard function based on warranty data.

3.6.4 Usage Mode

Products are often used intermittently, resulting in usage pattern such as that shown in Fig. 3.9. Intermittent usage involves a cyclic change from the "Operate" state to the "Idle" state in an uncertain manner. Here $\tilde{T}_{1j}$ denotes the time in operating state and $\tilde{T}_{0j}$ the time in the idle state during the jth cycle.

Let $R_0(t)$ denote the reliability of the product when it is used continuously and $R_i(t)$ the reliability when used intermittently. In order to link the two, we need to model operate and idle times.

Special Case We assume the following:

1. $\tilde{T}_{1j}$ is a sequence of independent and identically distributed (iid) random variables from a distribution $G_1(t)$
2. $\tilde{T}_{0j}$ is a sequence of iid random variables from a distribution $G_0(t)$
3. There is no degradation when an item is in its idle state

Then it can be shown [14] that

$$R_i(t) = R_0(t) + \int_0^t R_0(z)h(z,t)dz \tag{3.13}$$

where

$$h(z,t) = g_1(z)\bar{G}_0(t-z) + \int_0^z \left[\int_0^{t-x} h(z-x, t-x-y) g_0(y) dy \right] g_1(x) dx. \quad (3.14)$$

Since $h(z,t) > 0$, we have from (3.13) that $R_i(t) \geq R_0(t)$, as would be expected.

3.6.5 Usage Intensity (Operating Load)

A product is designed for some nominal usage intensity (for example, the number of washes per week and/or size of loads washed in a washing machine; the number of miles travelled per year in an automobile). Usage intensity can vary considerably across the customer population. When the usage intensity is higher (lower) than the nominal usage intensity, the degradation (due to higher wear and/or increased stresses on the components) is faster (slower). As a result, the actual field reliability can be lower or higher than the design reliability.[9]

We use the term "operating environment" to cover all of these. Let s denote the stress on the components in operation. Let s_0 denote the stress (electrical, mechanical and/or thermal, depending on the product) on the components under nominal usage intensity. Define $\tilde{s} = s/s_0$. Let $R_e(t)$ denote the field reliability (which takes into account the influence of the operating environment) and $R_0(t)$ the design reliability. The two well known models linking field reliability to design reliability are the following:

- Model 1: Accelerated Failure Time (AFT) Model [16]
- Model 2: Proportional Hazard (PH) Model [10]

3.6.5.1 AFT Model

Let T_s denote the time to failure under stress s and T_0 the failure time under nominal stress. The AFT model assumes the following

$$T_s = T_0 \phi(\tilde{s}) \quad (3.15)$$

where $\phi(\tilde{s})$ is a non-negative and monotonically increasing function with

$$\phi(\tilde{s}) \begin{cases} >1 & \text{when } \tilde{s} > 1 \\ =1 & \text{when } \tilde{s} = 1 \\ <1 & \text{when } \tilde{s} < 1 \end{cases} \quad (3.16)$$

[9] The same is true regarding the operating environment—for example, road conditions in the case of an automobile, operating temperature in the case of an electronic product.

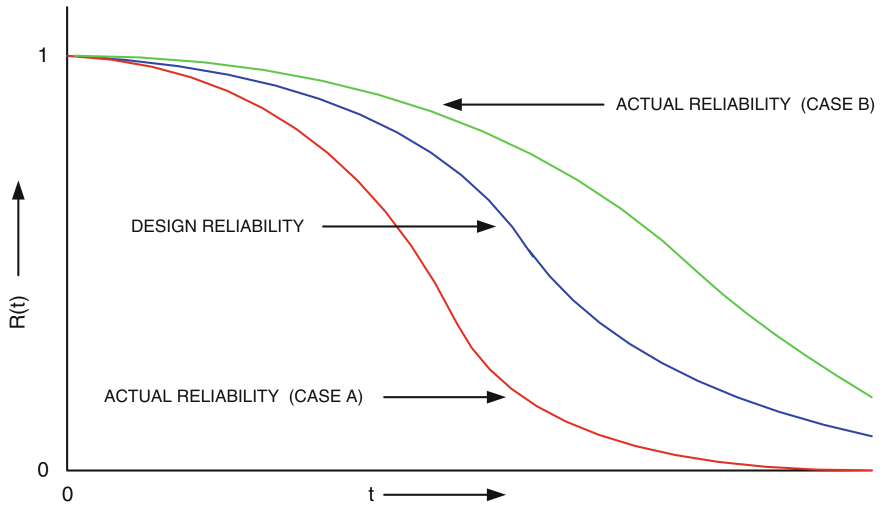


Fig. 3.10 Design and actual (field) reliabilities

As a result, $R_e(t)$ has the same form as $R_0(t)$ and the two scale parameters are linked by a relationship similar to that in (3.15). The scale parameter for $R_e(t)$ decreases [increases] as $\tilde{s}$ increases [decreases].

Figure 3.10 shows the effect of $\phi(\tilde{s})$ on the field reliability, with case A corresponding to $s > s_0$ and case B corresponding to $s < s_0$.

3.6.5.2 PH Model

Let $h_e(t)$ $[h_0(t)]$ denote the hazard function associated with $R_e(t)$ $[R_0(t)]$. The PH model assumes that

$$h_e(t) = h_0(t)\phi(\tilde{s}) \tag{3.17}$$

where $\phi(\tilde{s})$ is as in the AFT Model. As a result, $R_e(t) = [R_0(t)]^{\phi(\tilde{s})}$.

3.6.6 *Other Notions of Usage*

In addition to intermittent usage discussed in Sect. 3.6.4, one can define two other notions of usage, namely:

1. *Number of times an item is used*: Let $N(t)$ denote the number of times an item is used is over the interval $[0, t)$. Typical examples are (a) the landing gear used in the landing of an aircraft, and (b) number of loads done in a washing machine.

2. *Output of an item*: Let $U(t)$ denote the usage up to time t. The output is some measurable quantity. Typical examples of this are (a) miles an automobiles is driven, and (b) copies made on a photocopier.

In these cases, the item degradation and failure depend on the age and usage of the product. This can be modeled in several different ways. Approaches to modeling are discussed in Chap. 6.

3.7 Modeling Failures over Time

When a repairable item fails, it can either be repaired or replaced by a new item. In the case of a non-repairable item, the only option is to replace the failed item by a new one. Since failures occur in an uncertain manner, the number of failures over a time interval is a non-negative random variable. The distribution of this variable depends on the failure distribution of the item, the actions (repair or replace) taken after each failure, and the type of repair.

In this section, we model the number of failures over the interval $[0, t)$, starting with a new item at $t = 0$, for several different scenarios. Let $N(t)$ denote the number of failures over $[0, t)$. This is a counting process (see Appendix A). Let $p_j(t)$ denote the probability that $N(t) = j$, $j = 1, 2, \ldots$. Models for repairable and non-repairable items are as follows:

3.7.1 Non-Repairable Product

In the case of non-repairable product, every failure results in the replacement of the failed item by a new item. We assume that all new items are statistically similar, with distribution function $F(t)$. If the failures are detected and replaced immediately with replacement time negligible, then $N(t)$ is an ordinary renewal process, and we have the following results (see Appendix B):

$$p_j(t) = P\{N(t) = j\} = F^{(j)}(t) - F^{(j+1)}(t), \tag{3.18}$$

where $F^{(j)}(t)$ is the j-fold convolution of $F(t)$ with itself, and the expected number of failures over $[0, t)$ is given by

$$M(t) = F(t) + \int_0^t M(t - t')f(t')dt' \tag{3.19}$$

In general, it is difficult to obtain an analytical expression for $M(t)$ and computational approaches must be used to evaluate it [3].

3.7.2 Repairable Product

In this case, the characterization of the number of failures over time depends on the type of repair. The two types of repair are as follows:

3.7.2.1 Minimal Repair

Here the failure rate after repair is essentially the same as that if the item had not failed [1]. This is appropriate for complex products for which the product failure is due to failure of one or few of its components. The equipment becomes operational by replacing (or repairing) the failed components. This action ordinarily has very little impact on the reliability characteristics of the product.

If the failures are statistically independent, then $N(t)$ is a non-stationary Poisson process with intensity function $\lambda(t) = h(t)$, the failure rate associated with $F(t)$ [15]. As a result, we have the following (see Appendix B):

$$p_j(t) = P\{N(t) = j\} = \frac{e^{-\Lambda(t)}\{\Lambda(t)\}^j}{j!} \tag{3.20}$$

where

$$\Lambda(t) = \int_0^t \lambda(t')dt', \tag{3.21}$$

and the expected number of failures over $[0, t)$ is given by

$$E[N(t)] = \Lambda(t) \tag{3.22}$$

3.7.2.2 Imperfect Repair

Here the failure rate changes (in either direction) after repair. Many different types of imperfect repair models have been proposed [17]. The two that have been used extensively are the following:

Reduction in failure rate: If the repair time is negligible, then $h(t^+) = h(t^-) - \delta$, where t is the time at which the failure occurs and δ is the reduction, subject to the constraint $0 \le \delta < h(t^+) - h(0)$.

Reduction in age: This involves the notion of virtual age [9]. Let $A(t)$ denote the virtual age at time t. If the repair time is negligible, then $A(t^+) = A(t^-) - x$ if the failure occurs at time t and the reduction in age is x, subject to the constraint $0 \le x < A(t^-)$.

Comment: $\delta = 0$ and $x = 0$ imply minimal repair.

3.7.2.3 Repaired Items Different from New

Here, the failed item is subjected to a major overhaul which results in the failure distribution of the repaired items being $F_r(t)$, say, which is different from the failure distribution, $F(t)$, for new items. Since repaired items are assumed to be inferior to new ones, the mean time to failure for a repaired item is taken to be smaller than that for a new item.

3.8 Linking Product Reliability and Component Reliabilities

Even simple products are built using many components, and the number used increases with the complexity of the product. As such, a product can be viewed as a system of interconnected components. In Chap. 1, we discussed a decomposition of a product or system involving several levels. The number of levels that is appropriate depends on the product. The performance of the product depends on the state of the system (working, failed, or in one of several partially failed states) and this in turn depends on the state (working/failed) of the various components.

The two approaches for linking product reliability to component are (1) reliability block diagrams and (2) fault tree analysis. We discuss these briefly below. For additional details, see [4].

3.8.1 Reliability Block Diagrams

In a reliability block diagram, each component is represented by a block with two end points. When the component is in its working state, there is a connection between the two end points. This connection is broken when the component is in a failed state. A multi-component system can be represented as a network of such blocks, each with two end points. The system is in working state if there is a connected path between the two end points. If no such path exists, then the system is in a failed state. Systems may be of the following types:

Series Structure: This represents the case where the system is in its working state only when all the components are in working states.

Parallel Structure: This represents the case where the system is in a failed state only when all of the components are in failed states.

General Structure: This is a combination of series and parallel sub-structures and is needed for modeling more complex products.

3.8.2 Fault Tree Analysis (FTA)

A fault tree is a logic diagram that displays the relationship between a potential event affecting system performance and the reasons or underlying causes for this event. The reason may be failures (primary or secondary) of one or more components of the system, environmental conditions, human errors, and other factors.

A fault tree illustrates the state of the system (denoted the TOP event) in terms of the states (working/failed) of the system's components (denoted basic events). The connections are done using *gates*, where the output from a gate is determined by the inputs to it. A special set of symbols (for gates and basic events) is used for this purpose.[10]

3.8.3 Structure Function and Product Reliability

Let $X_i(t),\ 1 \leq i \leq n$, denote the state of component i, at time t, with

$$X_i(t) = \begin{cases} 1 & \text{if component } i \text{ is in working state at time } t \\ 0 & \text{if component } i \text{ is in failed state at time } t \end{cases} \tag{3.23}$$

Let $\underset{\sim}{X}(t) = (X_1(t), X_2(t), \ldots, X_n(t))$ denote the state of the n components at time t, and $X_S(t)$ (a binary random variable) denote the state of the system at time t. Then from FTA one can derive an expression of the form

$$X_S(t) = \phi(\underset{\sim}{X}(t)), \tag{3.24}$$

which links the component states to the system state. $\phi(\cdot)$ is called the structure function.[11]

Let $R_S(t)$ and $\underset{\sim}{R}(t) = (R_1(t), R_2(t), \ldots, R_n(t))$ denote the reliability of the system and of the set of reliabilities of the n components, respectively. If the component failures are independent, then

$$R_S(t) = \phi(\underset{\sim}{R}(t)) \tag{3.25}$$

so that we have the system reliability in terms of the component reliabilities. Results for the two simplest systems are:

Series Structure

$$R_S(t) = \prod_{i=1}^{n} R_i(t) \tag{3.26}$$

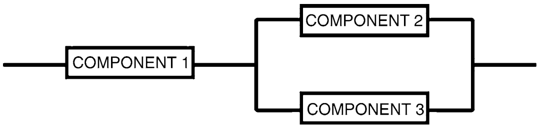
[10] For more on the construction and analysis of fault trees, see [4] and [18].

[11] The details can be found in many books on reliability; see, for example, [4, 18].

Parallel structure

$$R_S(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t)) \tag{3.27}$$

Example 3.3 Suppose a system is constructed based on three components as shown by the following diagram.



If the lifetimes of components 1, 2 and 3 all follow exponential distributions (A.22) with $\lambda = 0.001$, 0.002 and 0.003 failures per hour, respectively, then the reliability of the system for ten hours ($t = 10$) can be computed as follows:

Components 2 and 3 are a subsystem in parallel structure. The reliability of this subsystem at $t = 10$ (based on (3.27)) is

$$\begin{aligned} R_{2,3}(t = 10) &= 1 - \{(1 - R_2(10))(1 - R_3(10))\} = R_2(10) + R_3(10) - R_2(10)R_3(10) \\ &= e^{-0.002\times 10} + e^{-0.003\times 10} - e^{-0.002\times 10}e^{-0.003\times 10} = 0.99941 \end{aligned}$$

Component 1 and the sub-system with components 2 and 3 are in series structure. From (3.26), the reliability of the system at $t = 10$ is

$$R_s(t = 10) = R_1(10)R_{2,3}(10) = e^{-0.001\times 10} \times 0.99941 = 0.98947.$$

3.9 Warranty and Reliability

As mentioned in Chap. 1, offering warranty results in additional costs to the manufacturer. The various factors that affect these costs are shown in Fig. 3.11.

The key factors are:

1. Design reliability
2. Inherent reliability
3. Operating environment
4. Servicing strategy

In this chapter we have focused on (1)–(3). The effect of these on warranty costs are discussed in Chaps. 6 and 7.

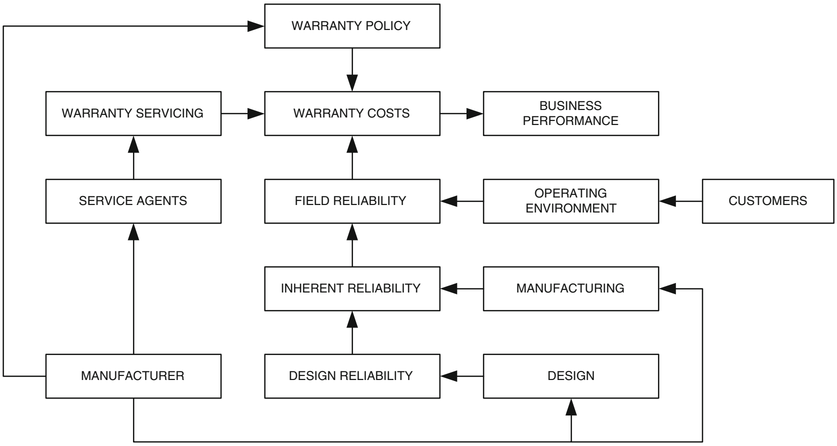


Fig. 3.11 Reliability and warranty

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