Inflation Derivatives

The market for financial inflation products started with public sector bonds linked to some measure for inflation of prices of (mainly) goods and services. This dates back to as early as the first half of the eighteenth century when the state of Massachusetts issued bonds linked to the price of silver on the London Exchange [4]. Over time, and particularly, in the last 20 years or so, the dominant index used for inflation-linked bonds has become the consumer price index (CPI). A notable exception is the UK inflation-indexed gilt market, which is linked to the retail price index (RPI).[a] The actual cash-flow structure of inflation-indexed bonds varies from issue to issue, including capital-indexed bonds (CIBs), *interest-indexed bonds*, *current pay bonds*, *indexed annuity bonds*, *indexed zero-coupon bonds*, and others. By far the most common cash-flow structure is the CIB, on which we shall focus in the remainder of this article.

Bonds, Asset Swaps, and the Breakeven Curve

Inflation-indexed bonds (of CIB type) are defined by

- N—a notional;
- I—the inflation index;
- L—a lag (often three months);
- $T_i : \{1 \le i \le n\}$—coupon dates;
- $c_i : \{1 \le i \le n\}$—coupon at date T_i (usually, all c_i are equal); and
- $I(T_0 - L)$—the bond's base index value.

The bond pays the regular coupon payments

$$N c_i \frac{I(T_i - L)}{I(T_0 - L)} \tag{1}$$

plus the inflation-adjusted final redemption, which often contains a capital guarantee according to

$$N \max\left(\frac{I(T_n - L)}{I(T_0 - L)}, 1\right) \tag{2}$$

Asset swap packages swapping the inflation bond for a floating leg are liquid in some markets such as for bonds linked to the CPTFEMU (also known as the *HICP*) index. Since the present value of an inflation-linked bond can be decomposed into the value of each coupon

$$P_{T_i}(T_0) N c_i \frac{1}{I(T_0 - L)} \mathsf{E}_0^{\mathcal{M}(P_{T_i})}[I(T_i - L)] \tag{3}$$

wherein $\mathsf{E}_\tau^{\mathcal{M}(X)}[\cdot]$ denotes expectation in filtration $\mathcal{F}_\tau$ under the measure induced by choosing X as numéraire, and the value of the final redemption

$$P_{T_n}(T_0) N \frac{1}{I(T_0 - L)} \times \mathsf{E}_0^{\mathcal{M}(P_{T_n})}[\max(I(T_n - L), I(T_0 - L))] \tag{4}$$

these products give us a mechanism to calibrate the *forward curve* $F(t, T)$, where

$$\begin{aligned} F(t, T - L) &:= \text{The index forward for (payment)} \\ &\qquad \text{time } T \text{ seen at time } t \\ &:= \mathsf{E}_t^{\mathcal{M}(P_T)}[I(T - L)] \end{aligned} \tag{5}$$

The forward curve is often also referred to as the *breakeven curve*. The realized inflation index fixing level is thus naturally $I(T) = F(T, T)$. Note that while equation (4), strictly speaking, requires a stochastic model for consistent evaluation due to the convexity of the $\max(\cdot, 1)$ function, in practice, the $\max(\cdot, 1)$ part is usually ignored, since its influence on valuation is below the level of price resolution.[b]

If there were a multitude of inflation-linked bonds, or associated asset swaps, with well-dispersed coupon dates liquidly available for any given inflation index, then the above argument would be all that is needed for the construction of a forward index curve, that is, breakeven inflation curve. In reality, though, for many inflation markets, there is only a small number of reasonably liquid bonds or asset swaps available. This makes it necessary to use interpolation techniques for forward inflation levels or rates between the attainable index-linked bond's maturity dates. In some cases, this may mean that for the construction of a 10-year (or longer) inflation curve, only three bonds are available, and extreme care must be taken for the choice of interpolation. However, even when a sufficiently large number of bonds is traded, to have a forward inflation rate for each year determined by the bond market, sophisticated interpolation methods are still needed. This is because of inflation's *seasonal* nature. For instance, consumer prices tend to go up significantly more than the annual average just before Christmas and tend to drop (or rise less than the annual average) just after.

The most common approach to incorporate seasonality into the breakeven curve is to analyze the statistical deviation of the month-on-month inflation rate from the annual average with the aid of historical data, and to overlay a seasonality adjustment on top of an annual inflation average curve in a manner such that, by construction, the annual inflation index growth is preserved. In addition, some authors used to suggest that one may want to add a long-term attenuation function (such as $e^{-\lambda t}$) for the magnitude of seasonality. This was supposed to represent the view that, since we have very little knowledge about long-term inflation seasonality, one may not wish to forecast any seasonality structure. This idea has gone out of fashion though, probably partly based on the realization that, historically, the seasonality of inflation became *more* pronounced over time, not exponentially *less*.

Inflation Derivatives

Daily Inflation Reference

Ultimately, all inflation fixings are based on the publication of a respective index level by a government or cross-government funded organization such as Eurostat for the HICP index series in Europe, or the Bureau of Labor Statistics for the CPI-U in the United States. This publication tends to be monthly, and usually on the same day of the month with a small amount of variability in the publication date. In most inflation bond markets, index-linked bonds are written on the publication of these published index levels in a straightforward manner such as $I(T_i)/I(T_0)$ times a fixed number as discussed in the previous section, with T_i indicating that a certain month's publication level is to be used. For some inflation bonds, however, the inflation reference level is not a single month's published fixing, but instead, an average over the two nearest fixings. In this manner, the fact that a bond's coupon is possibly paid between two index publication dates, and thus should really benefit from a value between the two levels, can be catered for. French OATi and OAT€i bonds, for instance, use this concept of the daily inflation reference (DIR) defined as follows:

$$\mathrm{DIR}(T) = I\left(T_{m(T)-3}\right) + \frac{n_{\mathrm{day}}(T) - 1}{n_{\mathrm{days}}(m(T))} \times \left[I\left(T_{m(T)-2}\right) - I\left(T_{m(T)-3}\right)\right] \quad (6)$$

with $m(T)$ indicating the month in which the reference date T lies, T_i the publication date of month i, $n_{\mathrm{day}}(T)$ the number of the day of date T in its month, and $n_{\mathrm{days}}(m(T))$ the number of days in the month in which T lies. For example, the DIR applicable to June 21st is $\frac{10}{30}$ times the HICP for March plus $\frac{20}{30}$ times the HICP for April. While the DIR is in itself not an inflation derivative, it is a common building block for derivatives in any market that uses the DIR in any bond coupon definitions. The DIR clause complicates the use of any model that renders inflation index levels as lognormal or similar, since any payoff depending on the DIR thus depends on the weighted sum of two index fixings.

Futures

Futures on the Eurozone HICP and the US CPI have been traded on the Chicago Mercantile Exchange since February 2004. Eurex launched Euro inflation futures based on the Eurozone HICP in January 2008. Both exchanges show, to date, very little actual trading activity in these contracts. An inflation futures contract settles at maturity at

$$M\left(1 - \frac{1}{\Omega}\left(\frac{I(T-L)}{I(T-L-\Omega)} - 1\right)\right) \quad (7)$$

with M being a contract size multiplier and Ω an additional time offset. The lag L is usually one month. The offset Ω is three months for CPI-U (also known as CPURNSA) on the CME, that is, $\Omega = 1/4$ above. For the HICP (also known as *CPTFEMU*) on both Eurex and CME, $\Omega = 1$, that is, one year. Exactly why the inflation trading community has paid little attention to these futures is not entirely clear, though, one explanation may be the difference in inflation linkage between bonds and futures. Both HICP-linked bonds and US Treasury-Inflation Protected Securities (TIPS) pay coupons on an inflation-adjusted notional, that is, they are CIBs. In contrast, both CPI and HICP futures pay period-on-period inflation rates. As a consequence, a futures-based inflation hedge of a single CIB coupon would require a whole sequence of futures positions and would leave the position still exposed to realized period-on-period covariance.

Zero-coupon Swaps

This simple inflation derivative is as straightforward as the simplest derivative in other asset classes: two